ARTICLE NO. 80-4042

Energy-Sink Analysis of Systems Containing Driven Rotors

Thomas R. Kane*
Stanford University, Stanford, Calif.
and
David A. Levinson‡
Lockheed Palo Alto Research Laboratory, Palo Alto, Calif.

The energy-sink method has come into widespread use as an analytical technique for obtaining descriptions of spacecraft attitude motions, even when a spacecraft contains driven rotors—devices that can act as energy sources. In this paper it is shown by means of examples that under these circumstances the energy-sink approach can lead to incorrect results.

Introduction

THE simulation of attitude motions of highly deformable, dissipative spacecraft is so difficult that one is highly motivated to resort to heuristic methods of analysis, particularly when the deformability and/or dissipation characteristics of the spacecraft are not well defined, as during preliminary design. One such method is the so-called energy-sink approach, which has been used extensively[1-26] to obtain qualitative and quantitative information regarding attitude motions of relatively simple spacecraft. However, the use of this technique is frequently accompanied by rather strong disclaimers,[11,12,15,19-21,24] which suggests that the designer is well-advised to view it with suspicion. More particularly, when the spacecraft under consideration contains driven rotors, as when momentum wheels or control moment gyros are employed for attitude control, one must entertain doubts regarding the validity of energy-sink arguments, for such components of a spacecraft can play the roles of energy *sources,* a fact first pointed out by Landon and Stewart.[13] Nevertheless, the method has been employed even under these circumstances[15-26]; and, although some published work contains material showing that incorrect conclusions can be reached in this way, failure of the energy-sink approach has not been the central issue in anything that has appeared in print to date, with the result that the dangers inherent in the use of the method remain obscure. The present paper is intended to remedy this situation.

This paper begins with the development of a formula that enables one to find the angle (nutation angle) between the central angular momentum vector and the symmetry axis of a torque-free, axisymmetric gyrostat as a function of the system's kinetic energy of rotation. A suitable kinetic energy time-history reflecting energy dissipation is then postulated, and the formula is used to construct a plot of the nutation angle as a function of time. To compare this energy-sink prediction with the actual motion of a deformable spacecraft, a gyrostat carrying a nutation damper is considered next. This system has deformability and energy dissipation capabilities of the kind one must deal with when analyzing large, flexible spacecraft, but it is sufficiently simple to permit the formulation of exact differential equations of motion. Predictions of the nutation angle time-history obtained from numerical solutions of these equations thus can be compared with the energy-sink predictions, and such comparisons show that the energy-sink method can lead to both quantitatively and qualitatively incorrect descriptions of attitude motions.

Received March 8, 1979; revision received Sept. 14, 1979.

Index categories: Spacecraft Dynamics and Control; Spacecraft Simulation.

*Professor of Applied Mechanics.

†Senior Associate Scientist. Member AIAA.

Analysis

Figure 1 represents a spacecraft S formed by a deformable, dissipative body A that carries a rigid, axisymmetric rotor B. B is made to rotate with a constant angular speed relative to bearings attached to A; when A is undeformed, the central inertia ellipsoid of S is a spheroid whose axis is parallel to that of B. S is presumed to be moving in a Newtonian reference frame N under the action of forces whose resultant moment about S^*, the mass center of S, is equal to zero, so that H, the angular momentum of S relative to S^*, remains fixed in N.

Suppose one wished to find the nutation angle θ between H and g_I, a unit vector parallel to the axis of B, as a function of time t. Clearly, one could not hope to solve this problem exactly in the absence of further information regarding the deformability and dissipation characteristics of A. But even if such information were available, it could be exceedingly difficult to solve the problem exactly. In either event, therefore, one might consider the energy-sink analysis that follows.

If A were undeformable, the angle between H and g_I would remain constant and would be given by[27]

$$\phi = \cos^{-1}[(J\omega + J_I\omega_I)H^{-1}] \qquad (0 \le \phi \le \pi) \tag{1}$$

where ϕ, J, ω, J_I, ω_I, and H are defined as follows: ϕ is written in place of θ (see Fig. 1) to make it clear that we are dealing with a rigid body in place of the deformable body A; J

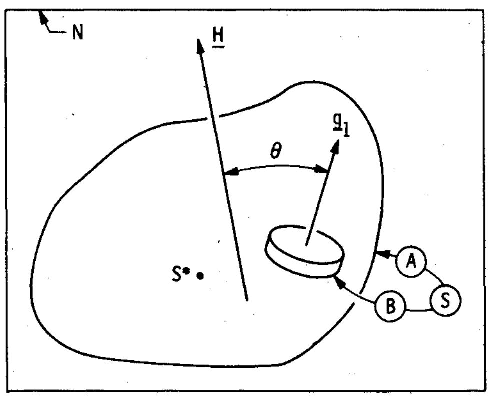

Fig. 1 Deformable spacecraft with rotor.

is the axial moment of inertia of B; $\omega \triangleq {}^A\omega^B . g_1$, where ${}^A\omega^B$ denotes the angular velocity of B in A; J_1 is the moment of inertia of S about a line passing through S^* and parallel to g_1; $\omega_1 \triangleq {}^N\omega^A . g_1$, where ${}^N\omega^A$ is the angular velocity of A in N; and H is the (constant) magnitude of $\boldsymbol{H}$. Furthermore, H would be given by

$$H = [(J\omega + J_1\omega_1)^2 + J_2^2(\omega_2^2 + \omega_3^2)]^{1/2} \tag{2}$$

where J_2 is the moment of inertia of S about any line passing through S^* and perpendicular to g_1; and $\omega_i \triangleq {}^N\omega^A \cdot a_i$ $(i = 2,3)$, where a_2 is a unit vector perpendicular to g_1, while $a_3 \triangleq g_1 \times a_2$. Finally, K, the rotational kinetic energy of S in N, could be expressed as

$$K = \tfrac{1}{2}[J\omega(\omega + 2\omega_1) + J_1\omega_1^2 + J_2(\omega_2^2 + \omega_3^2)] \tag{3}$$

Equations (2) and (3) can be used to eliminate ω_1 from Eq. (1) by proceeding as follows: solve Eq. (2) for $\omega_2^2 + \omega_3^2$, substitute into Eq. (3), solve the resulting quadratic equation for ω_1, and substitute into Eq. (1), obtaining

$$\phi = \cos^{-1}\left\{ \pm \left[\frac{\dfrac{(2K - J\omega^2)J_1 + (J\omega)^2}{H^2} - \dfrac{J_1}{J_2}}{1 - \dfrac{J_1}{J_2}} \right]^{1/2} \right\}$$

$$(J_1 \neq J_2; \quad 0 \leq \phi \leq \pi) \tag{4}$$

Deferring a discussion of the sign ambiguity in this expression, we note that K is a constant and that an energy-based approach to the solution of the problem originally posed, that is, the problem of finding θ when A is deformable, consists of using Eq. (4) to find ϕ (in place of θ), but treating K as a known function of t so as to account for the dissipative character of A. For instance, letting K_0 and K_1 denote the values of K at times t_0 and t_1, respectively, one can take

$$K = \frac{K_1 K_0 (t_1 - t_0)}{K_1 t_1 - K_0 t_0 - (K_1 - K_0)t} \tag{5}$$

in which event, if $t_1 > t_0 > 0$ and $K_0 > K_1$, a plot of K vs t appears as shown in Fig. 2. Suppose, for example, that $J_1 = 100$ kg m^2, $J_2 = 175$ kg m^2, $J = 1$ kg m^2, $\omega = 10$ rad/s, and for $t = 0$, ω_1 and ω_2 vanish while $\omega_3 = 1$ rad/s. Then, from Eq. (2), $H = 175.29$ N m s; and if t_0 and t_1 are arbitrarily assigned the values $t_0 = 0$ and $t_1 = 30$ s, then, from Eq. (3), $K_0 = 137.50$ N m, while the value of K_1 is a matter of conjecture. Assuming that K is a decreasing function of t (as it should be if the term "energy-sink" is to be taken at face value), we arbitrarily set $K_1 = 137.40$ N m. In accordance with Eq. (5), this means that we are postulating

$$K = \frac{5.6678 \times 10^6}{4.1220 \times 10^4 + t} \tag{6}$$

Finally, to achieve agreement between Eqs. (1) and (4) at $t = 0$, we choose the upper sign in Eq. (4), and we are now in a position to evaluate ϕ for $t > 0$, so long as $\cos\phi \neq 0$. Whenever $\cos\phi$ vanishes during the time interval $t_0 \leq t \leq t_1$, the sign ambiguity in Eq. (4) must be resolved anew. Consideration of the time-history of ϕ up to the instant in question and of time-derivatives of ϕ at this instant always permits one to select the proper sign. For $0 \leq t \leq 30$ s, we thus obtain the ϕ vs t plot shown in Fig. 3, which predicts that with increasing t, g_1 (see Fig. 1) will approach perpendicularity with $\boldsymbol{H}$. Moreover, since the ϕ vs t curve in Fig. 3 is very nearly linear, we can easily estimate the rate at which ϕ may be expected to grow: $\dot{\phi} \approx [\phi(30) - \phi(0)]/30 = (87.61 - 86.73)/30 = 2.943 \times 10^{-2}$ deg/s. The ease with which these qualitative and quantitative results are obtained is, of course, what makes the energy-sink approach so attractive.

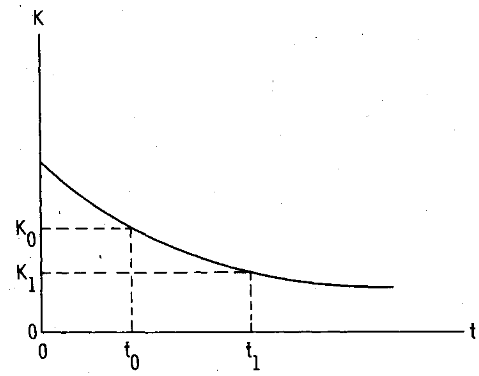

Fig. 2 Postulated time-history of kinetic energy.

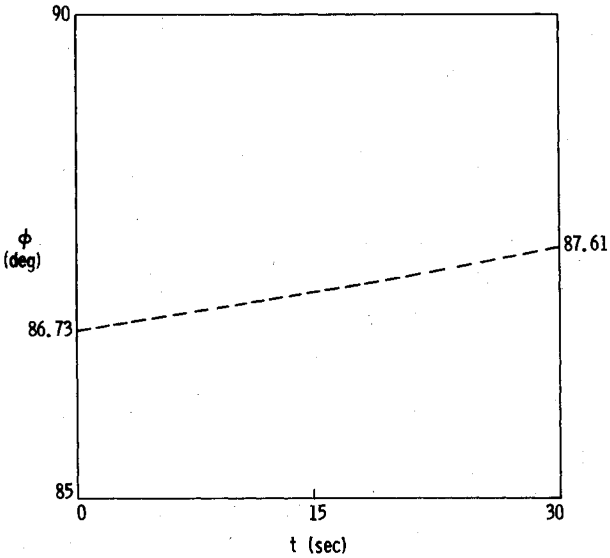

Fig. 3 Nutation angle time-history obtained from energy-sink analysis.

Now consider the special case of a body A (see Fig. 1) whose deformability and dissipation characteristics are known in all detail so that an exact determination of θ can be carried out. Specifically, let A be formed by a rigid body R that carries a particle P, the particle being free to move on a line L fixed in R, but being attached to R with a linear spring (spring constant σ) and a linear, viscous damper (damping constant δ). Furthermore, arrange these components as shown in Fig. 4, where G^* is the mass center of the gyrostat formed by R and B, G_1, G_2, G_3 are the central principal axes of inertia of G, and both L and G_1 are parallel to g_1. Finally, let the spring (and hence A) be undeformed when P lies on G_2 and choose b, the distance between L and G^*, m_P, the mass of P, m_G, the mass of G, and I_1, I_2, I_3, the moments of inertia of G about G_1, G_2, G_3, in such a way that

$$I_1 + \mu b^2 = J_1 \qquad I_2 = I_3 + \mu b^2 = J_2 \tag{7}$$

where μ is defined as

$$\mu \triangleq m_P m_G / (m_P + m_G) \tag{8}$$

If these requirements are fulfilled, it is guaranteed that when A is undeformed, the central inertia ellipsoid of S is a spheroid whose axis is parallel to g_1, the moment of inertia of S about the line passing through S^* and parallel to g_1 is equal to J_1, and the moment of inertia of S about any line passing through S^* and perpendicular to g_1 is equal to J_2. Fur-

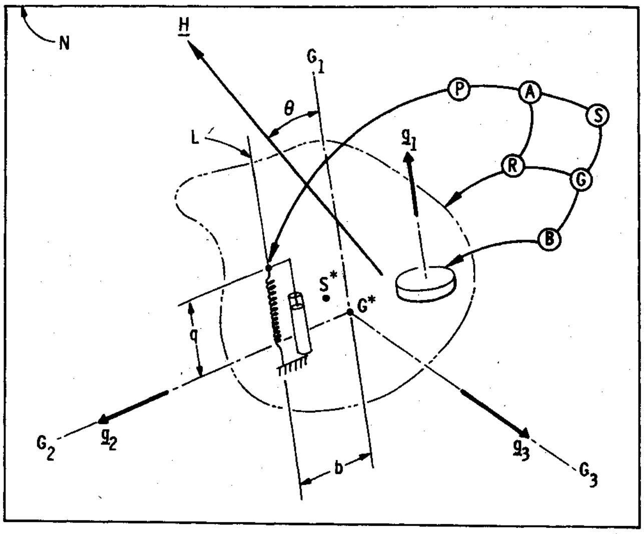

Fig. 4 Spacecraft with known deformability and dissipation characteristics.

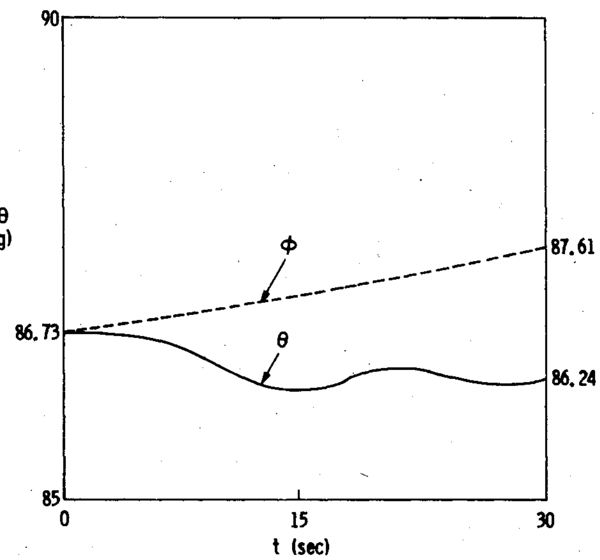

Fig. 5 Actual nutation angle time-history compared with energy-sink prediction.

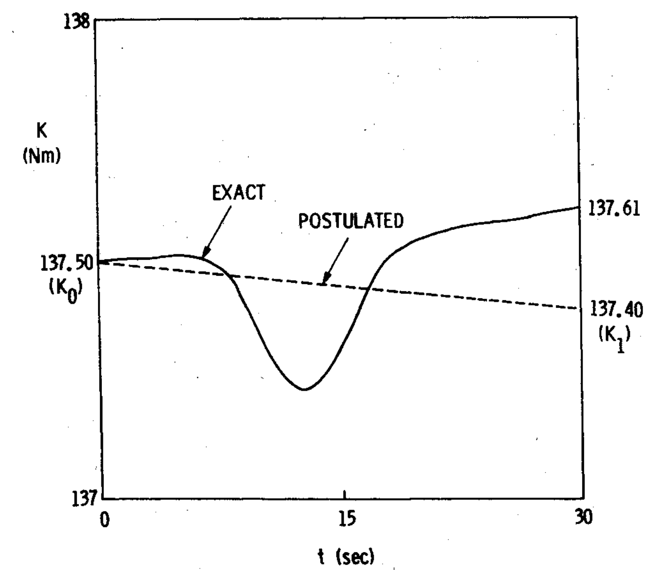

Fig. 6 Exact and postulated kinetic energy time-histories.

thermore, the angular momentum of S relative to S^* is then given, for all t, by

$$H = H_1 g_1 + H_2 g_2 + H_3 g_3 \tag{9}$$

provided H_1, H_2, H_3 be defined as

$$H_1 \triangleq J_1 \omega_1 - \mu b q \omega_2 + J\omega \tag{10}$$

$$H_2 \triangleq (J_2 + \mu q^2)\omega_2 - \mu b q \omega_1 \tag{11}$$

$$H_3 \triangleq (J_2 + \mu q^2)\omega_3 - \mu b \dot{q} \tag{12}$$

where q is the distance from P to G_2 (see Fig. 4) and $\omega_i = {}^N\omega^R \cdot g_i$ $(i=1,2,3)$. Consequently, θ (see Fig. 4) can be expressed as

$$\theta = \cos^{-1}(g_1 \cdot H/H) = \cos^{-1}[H_1(H_1^2 + H_2^2 + H_3^2)^{-1/2}]$$

$$(0 \leq \theta \leq \pi) \tag{13}$$

and one can now find θ exactly by using Eqs. (10-13) after solving the differential equations governing ω_1, ω_2, ω_3, and q, namely

$$J_1 \dot{\omega}_1 - \mu b q \dot{\omega}_2 + \mu b(\omega_3 \omega_1 q - 2\omega_2 \dot{q}) = 0 \tag{14}$$

$$\mu b q \dot{\omega}_1 - (J_2 + \mu q^2)\dot{\omega}_2 + \mu q(\omega_3 \omega_1 q + \omega_2 \omega_3 b - 2\omega_2 \dot{q})$$

$$+ (J_2 - J_1)\omega_3 \omega_1 - J\omega\omega_3 = 0 \tag{15}$$

$$(J_2 + \mu q^2)\dot{\omega}_3 - \mu b \ddot{q} + \mu[(\omega_2^2 - \omega_1^2)qb + \omega_1 \omega_2 q^2 + 2q\dot{q}\omega_3]$$

$$+ (J_2 - J_1)\omega_1 \omega_2 - J\omega\omega_2 = 0 \tag{16}$$

$$\mu b \dot{\omega}_3 - \mu \ddot{q} + \mu[q(\omega_2^2 + \omega_1^2) - \omega_1 \omega_2 b] - \sigma q - \delta \dot{q} = 0 \tag{17}$$

Of course, this cannot be done in closed form, but numerical solutions can be effected readily. For example, taking $m_P = 10$ kg, $m_G = 990$ kg, $b = 1$ m, $\sigma = 10$ N/m, $\delta = 2$ N s/m, $q = \dot{q} = 0$ at $t = 0$, $J_1 = 100$ kg m^2, $J_2 = 175$ kg m^2, $J = 1$ kg m^2, $\omega = 10$ rad/s, and $\omega_1 = \omega_2 = 0$, $\omega_3 = 1$ rad/s at $t = 0$ (the values of J_1, J_2, J, ω, ω_1, ω_2, ω_3 are the same as those used previously), one obtains the θ vs t plot shown as a solid curve in Fig. 5. The dashed curve is the ϕ vs t plot previously displayed in Fig. 3 and reproduced here to facilitate comparison.

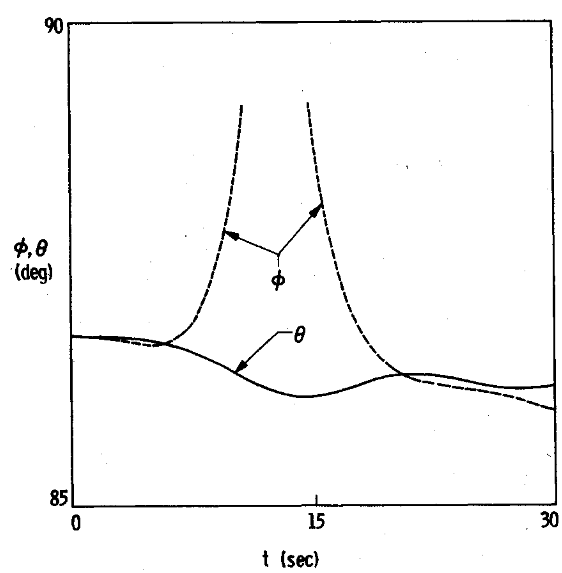

Fig. 7 Energy-sink analysis using exact kinetic energy time-history.

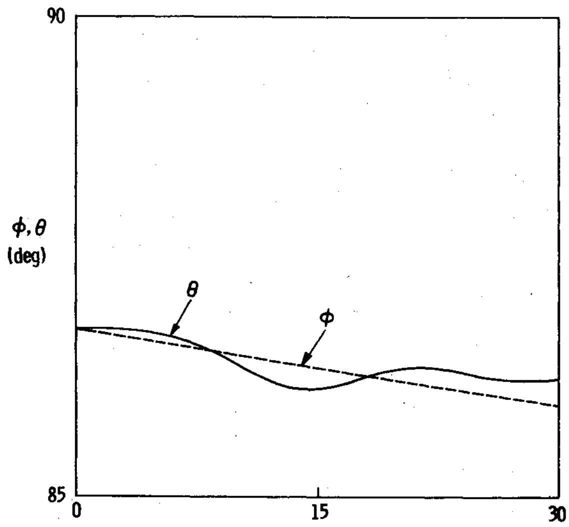

Fig. 8 Energy-sink analysis exploiting knowledge of exact kinetic energy time-history.

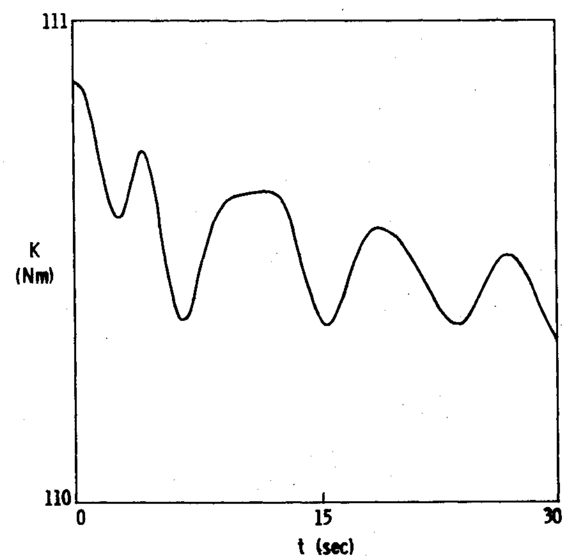

Fig. 9 Kinetic energy undergoing a net decrease with time.

Clearly, the two curves in Fig. 5 are in total disagreement with each other, both qualitatively and quantitatively. The basic reason underlying the discrepancy becomes apparent when one compares the K vs t curve corresponding to the solution of Eqs. (14-17) with the K vs t curve postulated in connection with the ϕ vs t curve of Fig. 5. Both K vs t curves are shown in Fig. 6. As can be seen, K_I not only has the wrong numerical value, but it stands in the wrong relationship to K_0: K_I should be larger than K_0, not smaller. That is, we have here an instance of a *net increase* in the kinetic energy of S, despite the fact that A is a dissipative body due to the presence of the viscous damper.

Lest it be thought that the failure just encountered is attributable to the fact that the energy changes associated with the motions of P and B relative to R were not modeled with sufficient accuracy because we employed the kinetic energy time-history given by Eq. (6), we now use the *true* time-history of K (the solid curve in Fig. 6) in conjunction with Eq. (4). This leads to the ϕ vs t curve shown together with the θ vs t curve in Fig. 7. Thus, it becomes evident that the method can lead to highly misleading results even when K is represented *exactly*. In fact, if a suitable value of K_I is used, Eq. (5) produces a better match between ϕ and θ than does the use of

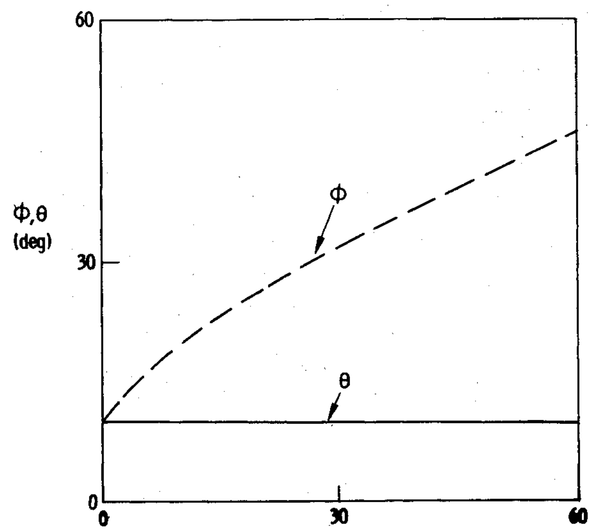

Fig. 10 Failure of energy-sink method for "non-flat" spin.

the exact K vs t curve. For example, with $K_I = 137.61$ N m (the actual value of K at $t = 30$ s), Eqs. (4) and (5) lead to Fig. 8. However, this observation does not support the notion that the energy method may yet prove efficacious, for how is one to know in advance, that is, in the absence of the exact solution, that one should use $K_I = 137.61$ N m? Indeed, can one even hope to know that K_I should be greater than K_0? To see that the answer is "no," take $\omega_1 = 1$ rad/s, $\omega_2 = 0.1$ rad/s, $\omega_3 = 0$ at $t = 0$, but leave all other values as before. In this event, solution of Eqs. (14-17) leads to the K vs t curve of Fig. 9, which reveals that the system under consideration is perfectly capable of performing motions during which K suffers a net decrease.

It should now be clear that energy-based analyses of the kind employed here cannot be trusted. Moreover, one gains additional support for this contention by making longer computer runs and by studying motions other than "flat spins." For example, Fig. 10 shows plots of ϕ and θ vs t for the same system considered in connection with Fig. 5, and for initial conditions differing from those used previously only in that ω_3 has the initial value 0.01 rad/s, rather than 1 rad/s. This leads to a motion during which θ remains at all times smaller than 10 deg, so that one is no longer dealing with a "flat spin." The values of K_0, K_I, t_0, and t_I [see Eq. (5)] used to produce the ϕ vs t curve [see Eq. (4)] are $K_0 = 50.009$ N m, $K_I = 49.900$ N m, $t_0 = 0$, and $t_I = 60$ s. (The present run is twice as long as each of the earlier ones.) Clearly, the disagreement between ϕ and θ is even more pronounced in Fig. 10 than in Fig. 5. Our conclusion is that, at least when dealing with systems containing driven rotors, one is well advised to undertake numerical solutions of the differential equations governing the motions of a perhaps relatively simple model of the system under consideration, rather than to use an energy-sink analysis.

Acknowledgments

This work was supported in part by the National Science Foundation under Grant No. NSF-ENG 77-04449 to Stanford University and in part by the Lockheed Missiles and Space Company Independent Research Program.

References

[1] Pilkington, W.C., "Vehicle Motions as Inferred from Radio-Signal-Strength Records," Jet Propulsion Laboratory, External Publication No. 551, Sept. 5, 1958.

[2] Bracewell, R.N. and Garriott, O.K., "Rotation of Artificial Earth Satellites," *Nature*, Vol. 182, Sept. 20, 1958, pp. 760-762.

[3] Bracewell, R.N., "Satellite Rotation," *Advances in the Astronautical Sciences,* Vol. 4, 1958, pp. 317-328.

[4] Carrier, G.F. and Miles, J.W., "On the Annular Damper for a Freely Precessing Gyroscope," *Journal of Applied Mechanics,* June 1960, pp. 237-240.

[5] Fitzgibbon, D.P., and Smith, W.E., "Final Report on Study of Viscous Liquid Passive Wobble Dampers for Spinning Satellites," Space Technology Laboratories, Inc., Report No. 8926-0008-MU-000, June 26, 1961.

[6] Haseltine, W.R., "Passive Damping of Wobbling Satellites: General Stability Theory and Example," *Journal of the Aerospace Sciences,* Vol. 29, No. 5, May 1962, pp. 543-550.

[7] Thomson, W.T. and Reiter, G.S., "Motion of an Asymmetric Spinning Body with Internal Dissipation," *AIAA Journal,* Vol. 1, June 1963, pp. 1429-1430.

[8] Miles, J.W., "On the Annular Damper for a Freely Precessing Gyroscope-II," *Journal of Applied Mechanics,* June 1963, pp. 189-192.

[9] Alper, J.R., "Analysis of Pendulum Damper for Satellite Wobble Damping," *Journal of Spacecraft and Rockets,* Vol. 2, 1965, pp. 50-54.

[10] Bhuta, P.G. and Koval, L.R., "Decay Rates of a Passive Precession Damper and Bounds," *Journal of Spacecraft and Rockets,* Vol. 3, March 1966, pp. 335-338.

[11] Bhuta, P.G. and Koval, L.R., "A Viscous Ring Damper for a Freely Precessing Satellite," *International Journal of Mechanical Sciences,* Vol. 8, 1966, pp. 383-395.

[12] Alfriend, K.T., "Analysis of a Partially Filled Viscous Ring Damper," Goddard Space Flight Center, NASA TM X-732-71-456, Oct. 1971.

[13] Landon, V.D. and Stewart, B., "Nutational Stability of an Axisymmetric Body Containing a Rotor," *Journal of Spacecraft and Rockets,* Vol. 1, Nov.-Dec. 1964, pp. 682-684.

[14] Taylor, R.S., and Conway, J.J., "Viscous Ring Precession Damper for Dual-Spin Spacecraft," *Proceedings of the Symposium on Attitude Stabilization and Control of Dual-Spin Spacecraft,* El Segundo, Calif., Aug. 1967, pp. 75-80.

[15] Likins, P.W., "Attitude Stability Criteria for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 4, Dec. 1967, pp. 1638-1643.

[16] Iorillo, A.J., "Hughes Gyrostat System," *Proceedings of the Symposium on Attitude Stabilization and Control of Dual-Spin Spacecraft,* El Segundo, Calif., Aug. 1967, pp. 257-266.

[17] Velman, J.R. and Belardi, J.W., "Gyrostat Attitude Dynamics," Hughes Aircraft Company, Report No. SSD 90154R, May 1969.

[18] Lim, W.K., "Passive Damping of the Forced Precession Motion of a Two-Body Satellite," *Journal of Spacecraft and Rockets,* Vol. 8, Jan. 1971, pp. 41-47.

[19] Scher, M.P., "Effects of Flexibility in the Bearing Assemblies of Dual-Spin Spacecraft," *AIAA Journal,* Vol. 9, May 1971, pp. 900-905.

[20] Likins, P.W., Tseng, G.T., and Mingori, D.L., "Stable Limit Cycles Due to Nonlinear Damping in Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 8, June 1971, pp. 568-574.

[21] Spencer, T.M., "Energy-Sink Analysis for Asymmetric Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 11, July 1974, pp. 463-468.

[22] Tonkin, S.W., "Non-Active Nutation Damping for Single-Spin Spacecraft of All Mass Distributions," *The Aeronautical Journal,* Sept. 1976, pp. 394-401.

[23] Kaplan, M.H., *Modern Spacecraft Dynamics and Control,* Wiley, New York, 1976, pp. 185-188.

[24] Mingori, D.L., "Effects of Energy Dissipation on the Attitude Stability of Dual-Spin Satellites," *AIAA Journal,* Vol. 7, Jan. 1969, pp. 20-27.

[25] Cloutier, G.J., "Nutation Damper Instability on Spin-Stabilized Spacecraft," *AIAA Journal,* Vol. 7, Nov. 1969, pp. 2110-2115.

[26] Spencer, T.M., "Cantilevered-Mass Nutation Damper for a Dual-Spin Spacecraft," *Proceedings of the Symposium on Attitude Stabilization and Control of Dual-Spin Spacecraft,* El Segundo, Calif., Aug. 1-2, 1967; Aerospace Report No. TR-0158 (3307-01)-16, pp. 91-109.

[27] Kane, T.R., "Solution of the Equations of Rotational Motion for a Class of Torque-Free Gyrostats," *AIAA Journal,* Vol. 8, June 1970, pp. 1141-1143.